Code :9F00104

MCA I Semester Regular & Supplementary Examinations, February 2011
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
(For students admitted in 2009 & 2010 only)

Time: 3 hours **Max Marks: 60**

Answer any FIVE questions
All questions carry equal marks
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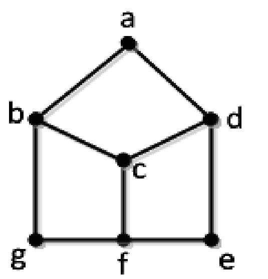
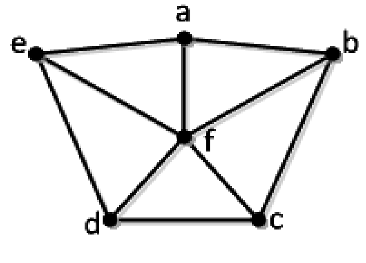
1. (a) What is a tautology? Prove that the following formula is a tautology.
$(((P \vee \neg Q) \rightarrow R) \leftrightarrow S) \vee \neg (((P \vee \neg Q) \rightarrow R) \leftrightarrow S)$
   (b) What is a principal disjunctive normal form? Obtain principal disjunctive normal form of
$P \rightarrow ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$
2. (a) Show that
$(x)(p(x) \rightarrow Q(x)) \wedge (x)\ (Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \Rightarrow R(x))$
   (b) Explain about free and bound variables in detail in the context of predicate logic.
3. (a) Explain about the following properties of a binary relation in a set. Give one example for each.
(i) Reflexive (ii) Symmetric (iii) Transitive (iv) Irreflexive (v) Antisymmetric
   (b) Define a partial order relation. Give an example. Let A be the set of factors of a particular positive integer m and let $\leq$ be the relation divides i.e.
$\leq = \{<x,y> / x \in A \wedge y \in A \wedge (x \text{ divides } y)\}$
Draw Hasse diagrams for (i) m=2 (ii) m=45.
4. (a) Define homomorphism of semigroups. Let $(s,*), (T,\triangle)$ *and* $(V,\oplus)$ be semigroups and $g : s \rightarrow T$ *and* $h : T \rightarrow V$ be semigroup homorphisms. Then prove that $(hog) : S \rightarrow V$ is a semi group homomorphism from $(S,*)$ *to* $(V,\oplus)$.
   (b) What is a monoid? Let S be a non empty set and p(s) be its power set. Prove that the algebra $< P(s), U >$ is a monoid.
5. (a) How many different license plates are there (allowing repetetions):
(i) involving 3 letters and 4 digits if the 3 letters must appear together either at the beginning or at the end of the plate?
(ii) involving 1,2 or 3 letters and 1,2,3 or 4 digits if the letters must occur together?
   (b) Use the binomial theorem to prove that $3^n = \sum_{r=0}^{n} C(n,r)2^r$
6. Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0\ for\ n \geq 2$
7. (a) Explain about different ways of representing a graph.
   (b) What is a spanning tree? Explain any one method for finding out spanning tree of a given graph with an example.
8. (a) Prove that there is no Hamiltonian cycle in the following graph.



   (b) Define chromatic number of a graph.Find the chromatic number of the following wheel graph.



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